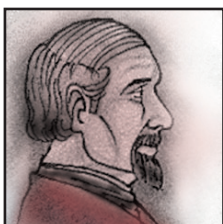


Chapter 2

Complex Numbers

“Imaginary numbers are a fine and wonderful refuge of the divine spirit almost an amphibian between being and non-being.”

- Gottfried Leibniz



Rafael Bombelli
(1526-1572)

Many mathematicians contributed to the full development of complex numbers. The rules for addition, subtraction, multiplication, and division of complex numbers were developed by the Italian mathematician Rafael Bombelli. He is generally regarded as the first person to develop an algebra of complex numbers. In honour of his accomplishments, a moon crater was named Bombelli.

Real Life Context

Complex numbers are useful in representing a phenomenon that has two parts varying at the same time, for instance an alternating current. Engineers, doctors, scientists, vehicle designers and others who use electromagnetic signals need to use complex numbers for strong signal to reach its destination. Complex numbers have essential concrete applications in signal processing, control theory, electromagnetism, fluid dynamics, quantum mechanics, cartography, and vibration analysis.



Learning Objectives

Upon completion of this chapter, students will be able to:

- perform algebraic operations on complex numbers
- plot the complex numbers in Argand plane
- find the conjugate and modulus of a complex number
- find the polar form and Euler form of a complex number
- apply de Moivre theorem to find the n^{th} roots of complex numbers.

2.1 Introduction to Complex Numbers

Before introducing complex numbers, let us try to answer the question “Whether there exists a real number whose square is negative?” Let’s look at simple examples to get the answer for it. Consider the equations 1 and 2.

Equation 1	Equation 2
$x^2 - 1 = 0$ $x = \pm\sqrt{1}$ $x = \pm 1$	$x^2 + 1 = 0$ $x = \pm\sqrt{-1}$ $x = \pm ?$