

Chapter 3

Theory of Equations



"It seems that if one is working from the point of view of getting beauty in one's equation, and if one has really a sound insight, one is on a sure line of progress."

- Paul Dirac

3.1 Introduction

One of the oldest problems in mathematics is solving algebraic equations, in particular, finding the roots of polynomial equations. Starting from Sumerian and Babylonians around 2000 BC (BCE), mathematicians and philosophers of Egypt, Greece, India, Arabia, China, and almost all parts of the world attempted to solve polynomial equations.

The ancient mathematicians stated the problems and their solutions entirely in terms of words. They attempted particular problems and there was no generality. Brahmagupta was the first to solve quadratic equations involving negative numbers. Euclid, Diophantus, Brahmagupta, Omar Khayyam, Fibonacci, Descartes, and Ruffini were a few among the mathematicians who worked on polynomial equations. Ruffini claimed that there was no algebraic formula to find the solutions to fifth degree equations by giving a lengthy argument which was difficult to follow; finally in 1823, Norwegian mathematician Abel proved it.



Abel
(1802-1829)



Suppose that a manufacturing company wants to pack its product into rectangular boxes. It plans to construct the boxes so that the length of the base is six units more than the breadth, and the height of the box is to be the average of the length and the breadth of the base. The company wants to know all possible measurements of the sides of the box when the volume is fixed.

If we let the breadth of the base as x , then the length is $x+6$ and its height is $x+3$. Hence the volume of the box is $x(x+3)(x+6)$. Suppose the volume is 2618 cubic units, then we must have $x^3 + 9x^2 + 18x = 2618$. If we are able to find an x satisfying the above equation, then we can construct a box of the required dimension.

We know a circle and a straight line cannot intersect at more than two points. But how can we prove this? Mathematical equations help us to prove such statements. The circle with centre at origin and radius r is represented by the equation $x^2 + y^2 = r^2$, in the xy -plane. We further know that a line, in the same plane, is given by the equation $ax + by + c = 0$. The points of intersection of the circle and the straight line are the points which satisfy both equations. In other words, the solutions of the simultaneous equations

$$x^2 + y^2 = r^2 \text{ and } ax + by + c = 0$$

give the points of intersection. Solving the above system of equations, we can conclude whether they touch each other, intersect at two points or do not intersect each other.