

Chapter 4

Inverse Trigonometric Functions



*“The power of Mathematics is often to change one thing into another,
to change geometry into language”*

- Marcus du Sautoy

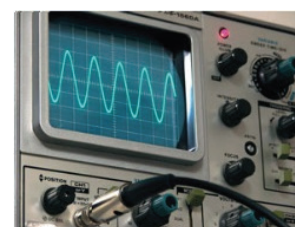
4.1 Introduction

In everyday life, indirect measurement is used to obtain solutions to problems that are impossible to solve using measurement tools. Trigonometry helps us to find measurements like heights of mountains and tall buildings without using measurement tools. Trigonometric functions and their inverse trigonometric functions are widely used in engineering and in other sciences including physics. They are useful not only in solving triangles, given the length of two sides of a right triangle, but also they help us in evaluating a certain type of integrals, such as $\int \frac{1}{\sqrt{a^2 - x^2}} dx$ and $\int \frac{1}{x^2 + a^2} dx$. The symbol $\sin^{-1} x$ denoting the inverse trigonometric function arcsine (x) of sine function was introduced by the British mathematician **John F.W.Herschel (1792-1871)**. For his work along with his father, he was presented with the Gold Medal of the Royal Astronomical Society in 1826.



John F.W. Herschel

An oscilloscope is an electronic device that converts electrical signals into graphs like that of sine function. By manipulating the controls, we can change the amplitude, the period and the phase shift of sine curves. The oscilloscope has many applications like measuring human heartbeats, where the trigonometric functions play a dominant role.



Let us consider some simple situations where inverse trigonometric functions are often used.

Illustration-1 (Slope problem)

Consider a straight line $y = mx + b$. Let us find the angle θ made by the line with x -axis in terms of slope m . The slope or gradient m is defined as the rate of change of a function, usually calculated by $m = \frac{\Delta y}{\Delta x}$. From right triangle (Fig. 4.1), $\tan \theta = \frac{\Delta y}{\Delta x}$. Thus, $\tan \theta = m$. In order to solve for θ , we need the inverse trigonometric function called **“inverse tangent function”**.

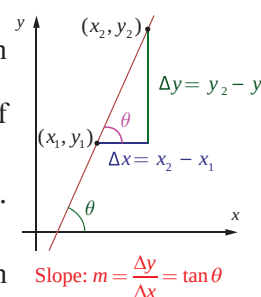


Fig. 4.1