

Chapter 6

Applications of Vector Algebra

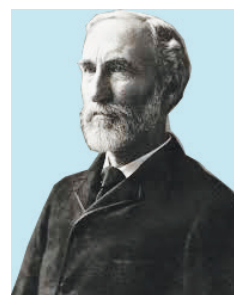


*“Mathematics is the science of the connection of magnitudes.
Magnitude is anything that can be put equal or unequal to another thing.
Two things are equal when in every assertion each may be replaced by the other.”*

— Hermann Günther Grassmann

6.1 Introduction

We are familiar with the concept of vectors, (*vectus* in Latin means “to carry”) from our XI standard text book. Further the modern version of Theory of Vectors arises from the ideas of Wessel (1745-1818) and Argand (1768-1822) when they attempt to describe the complex numbers geometrically as a directed line segment in a coordinate plane. We have seen that **a vector has magnitude and direction** and **two vectors with same magnitude and direction regardless of positions of their initial points are always equal**.



Josiah Willard Gibbs
(1839 – 1903)

We also have studied addition of two vectors, scalar multiplication of vectors, dot product, and cross product by denoting an arbitrary vector by the notation $\vec{a}$ or $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$. To understand the direction and magnitude of a given vector and all other concepts with a little more rigor, we shall recall the geometric introduction of vectors, which will be useful to discuss the equations of straight lines and planes. Great mathematicians Grassmann, Hamilton, Clifford and Gibbs were pioneers to introduce the dot and cross products of vectors.

The vector algebra has a few direct applications in physics and it has a lot of applications along with vector calculus in physics, engineering, and medicine. Some of them are mentioned below.

- To calculate the volume of a parallelepiped, the scalar triple product is used.
- To find the work done and torque in mechanics, the dot and cross products are respectively used.
- To introduce curl and divergence of vectors, vector algebra is used along with calculus. Curl and divergence are very much used in the study of electromagnetism, hydrodynamics, blood flow, rocket launching, and the path of a satellite.
- To calculate the distance between two aircrafts in the space and the angle between their paths, the dot and cross products are used.
- To install the solar panels by carefully considering the tilt of the roof, and the direction of the Sun so that it generates more solar power, a simple application of dot product of vectors is used. One can calculate the amount of solar power generated by a solar panel by using vector algebra.
- To measure angles and distance between the panels in the satellites, in the construction of networks of pipes in various industries, and, in calculating angles and distance between beams and structures in civil engineering, vector algebra is used.