

Chapter

7

Applications of Differential Calculus



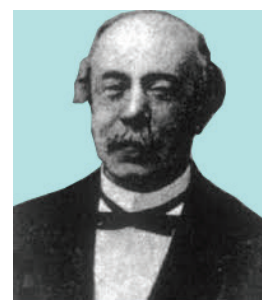
*“Nothing takes place in the world
whose meaning is not that of some maximum or minimum”*

- Leonhard Euler

7.1 Introduction

7.1.1 Early Developments

The primary objective of differential calculus is to partition something into smaller parts (infinitesimal parts), in order to determine how it changes. For this reason today's differential calculus was earlier named as **infinitesimal calculus**. Applications of differential calculus to problems in physics and astronomy was contemporary with the origin of science. All through the 18th century these applications were multiplied until Laplace and Lagrange, towards the end of the 18th century, had brought the whole range of the study of forces into the realm of analysis.



Rudolf Otto Sigismund Lipschitz
1832-1903

The development of applications of differentiation are also due to Lejeune Dirichlet, Riemann, von Neumann, Heine, Kronecker, Lipschitz, Christoffel, Kirchhoff, Beltrami, and many of the leading physicists of the century.

- Differential calculus has applications in geometry and dynamics.
- Derivatives of function, representing cost, strength, materials in a process, profit, etc., are used to determine the monotonicity of functions and there by to determine the extreme values of the quantities represented by those functions.
- Derivatives of a function do find a prominent place in many of the modelling problems in engineering and sciences.
- Differential calculus has applications in social sciences and medical sciences too.

Using just the first two derivatives of a function $f(x)$, in this chapter, the nature of the function, sketching of curve $y = f(x)$, and local extrema (maxima or minima) of $f(x)$ are determined. Further, using certain higher derivatives of $f(x)$ (if they exist), series expansion of $f(x)$ about a point are also discussed.



Learning Objectives

Upon completion of this chapter, students will be able to

- apply derivatives to geometrical problems
- use derivatives to physical problems
- identify the nature of curves like monotonicity, convexity, and concavity
- model real time problems for computing the extreme values using derivatives
- trace the curves for polynomials and other functions.