

Chapter 8

Differentials and Partial Derivatives



“He who hasn’t tasted bitter things hasn’t earned sweet things”

- Gottfried Wilhelm Leibniz

8.1 Introduction

Motivation

In real life we have to deal with many functions. Many times we have to estimate the change in the function due to change in the independent variable. Here are some real life situations.

- Suppose that a thin circular metal plate is heated uniformly. Then its radius increases and hence its area also increases. Suppose we can measure the approximate increase in the radius. How can we estimate the increase in the area of a circular plate?
- Suppose water is getting filled in water tank that is in the shape of an inverted right circular cone. In this process the height of the water level changes, the radius of the water level changes and the volume of the water in the tank changes as time changes. In a small interval of time, if we can measure the change in the height, change in the radius, how can we estimate the change in the volume of the water in the interval?
- A satellite is launched into the space from a launch pad. A camera is being set up, to observe the launch, at a safe distance from the launch pad. As the satellite lifts up, camera’s angle of elevation changes. If we know the two consecutive angles of elevation, within a small interval of time, how can we estimate the distance traveled by the satellite during that short interval of time?

To address these type of questions, we shall use the ideas of derivatives and partial derivatives to find linear approximations and differentials of the functions involved.

In the earlier chapters we have learnt the concept of derivative of a real-valued function of a single real variable. We have also learnt its applications in finding extremum of a function on its domain, and sketching the graph of a function. In this chapter, we shall see one more application of the derivative in estimating values of a function at some point. We know that linear functions, $y = mx + b$, are easy to work with; whereas nonlinear functions are computationally a bit tedious to work with.



Gottfried W Leibniz
(1646 - 1716)