

Chapter 10

Ordinary Differential Equations



“Mathematics is the most beautiful and most powerful creation of the human spirit”

– Stefan Banach

10.1 Introduction

Motivation and Early Developments

Just we look at some real life situations where

- the motion of projectile, rocket, satellite and planets
- the charge or current in the electric circuit
- the conduction of heat on a rod or in a slab
- the vibrations of a wire or membrane etc

are to be determined. The mathematical formulations of such problems emerge as differential equations under certain scientific laws. These laws involve various rates of change (derivatives) of one or more quantities with respect to other quantities. Thus the scientific laws manifest as mathematical equations involving derivatives, viz. differential equations.

Differential Equations emanate from the problems in geometry, mechanics, physics, chemistry, and engineering studies. We have studied about “rates” in our early classes. This is also known as instantaneous rate of change which is denoted as $\frac{dy}{dx}$.

We give below some relations between the rate of change and unknown functions that occur in real life situations.

- (a) The rate of change of y with respect to x is directly proportional to y :

$$\frac{dy}{dx} = ky.$$

- (b) The rate of change of y with respect to x is directly proportional to the product of y^2 and x :

$$\frac{dy}{dx} = ky^2x.$$

- (c) The rate of change of y with respect to x is inversely proportional to y :

$$\frac{dy}{dx} = \frac{k}{y}.$$

- (d) The rate of change of y with respect to x is directly proportional to y^2 and inversely proportional to $\sqrt{x}$:

$$\frac{dy}{dx} = k \frac{y^2}{\sqrt{x}}.$$

A differential equation is an equation in which some derivatives of the unknown function occur. In many cases the independent variable is taken to be time.